Model independent shape analysis of correlations
in 1, 2 or 3 dimensions

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Abstract

A generic, model-independent method for the analysis of the two-particle short-range correlations is presented, that can be utilized to describe e.g. Bose-Einstein (HBT or GGLP), statistical, dynamical or other short-range correlation functions. The method is based on a data-motivated choice for the zero-th order approximation for the shape of the correlation function, and on a systematic determination of the correction terms with the help of complete orthonormal sets of functions. The Edgeworth expansion is obtained for approximately Gaussian, the Laguerre expansion for approximately exponential correlation functions. Multi-dimensional expansions are also introduced and discussed.

Key words: correlations, elementary particle, heavy ion, statistical analysis

1 Introduction

Can one *model-independently* characterize the shape of two-particle correlation functions? In the present discussion, we address such model-independent characterizations of short-range correlations on the level of statistical analysis. We do not make any theoretical assumptions, neither on the thermal or non-thermal nature of the particle emitting source, nor on the negligibility of Coulomb and other final state interactions, nor on the presence or the negligibility of a coherent component in the source, nor on the negligibility of higher order quantum statistical symmetrization effects, nor on the negligibility of dynamical effects (e.g. fractal structure of gluon-jets) on the short-range part

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of the correlation function. We simply propose an expansion technique based on complete orthonormal sets of functions and an experimental choice for the zero-th order approximation to the BECF.

Although we do not assume here any theoretical knowledge about the short-range part of the two-particle correlation function, we assume the following

experimental properties:

i) The correlation function tends to a constant for large values of the relative momentum Q.

ii) The correlation function has a non-trivial structure at a certain value of its argument.

The location of the non-trivial structure in the correlation function is assumed for simplicity to be close to $Q = 0$.

The properties *i)* and *ii)* are well satisfied by e.g. the conventionally used two-particle Bose-Einstein correlation functions (BECF-s). For an introduction to and a review of the recently determined non-ideal features (e.g. non-Gaussian shapes in multi-dimensional Bose-Einstein correlation studies), we recommend ref. [1].

One of the parameters of the fit functions is the intercept parameter λ_*, that carries important information [2,3] on the possible restoration of $U_A(1)$ symmetry in hot and dense matter, when the core-halo picture can be applied [4]. However, the intercept parameter λ_* is known to be particularly sensitive to the chosen form of the parameterization of the correlation function. For an exponential fitting function, the extrapolated intercept is typically higher than that of the Gaussian fit, sometimes even by a factor of two. However, both parameters have only a *few percent* error from the minimization procedure [5,6].

It is well known [1], that certain 1 or 2 dimensional correlation functions can be fitted with both exponential and Gaussian forms. In case of the NA22 [7] and the UA1 data [8], the second-order correlation function exhibits a stronger than Gaussian or exponential rise at low values of the invariant relative momentum Q_I. Recently, even singular parameterizations were shown to represent well the precision data of the NA22 and the UA1 collaborations [7,8]. In this case, a formal interpolation of the fitting form to zero relative momentum would yield infinite value for the intercept. Furthermore, the intercept parameter λ_* of the core-halo picture is rather difficult to measure, as various non-ideal effects due to detector resolution, particle mis-identification, resonance decays, details of the Coulomb and strong final state interactions etc may influence this parameter of the fit. One should also mention, that if all of these difficulties are properly handled, the intercept parameter λ_* for like-sign

charged bosons is (usually) not larger, than unity as a consequence of quantum statistics for chaotic sources, even with a possible admixture of a coherent component. However, attractive final state interactions, fractal branching processes of gluon jets, or the appearance of one-mode or two-mode squeezed states [9,10] in the particle emitting source might provide arbitrarily large values for the intercept parameter.

Let us emphasize, that our new method is based *only* on the properties *i)* and *ii)* of the correlation function. Thus, the method is *really* model-independent, and it can be applied not only to Bose-Einstein correlation functions but to any other experimentally determined function, that satisfies properties *i)* and *ii)*.

2 Shape expansions using complete orthonormal sets of functions

Let us define the two-particle correlation function as the ratio of the two-particle invariant momentum distribution to the product of the single-particle invariant momentum distributions [11]:

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{N_2(\mathbf{k}_1, \mathbf{k}_2)}{N_1(\mathbf{k}_1)\, N_1(\mathbf{k}_2)}, \tag{1}$$

where $\mathbf{k}$ stands for the three-momentum of the detected particle of mass m and energy $E = \sqrt{m^2 + \mathbf{k}^2}$. For simplicity, we assume that at large relative momenta, the two-particle correlation function tends to 1; i.e. we normalize C_2 in such a manner that the possibly existing long-range correlations are removed by the normalization, which can be achieved by introducing an appropriate normalization coefficient into eq. (1). The two-particle correlator is introduced as

$$R_2(\mathbf{k}_1, \mathbf{k}_2) = C_2(\mathbf{k}_1, \mathbf{k}_2) - 1. \tag{2}$$

We assume that $R_2(\mathbf{k}_1, \mathbf{k}_2)$ is experimentally determined and it satisfies properties *i)* and *ii)*. Further, we assume that the measured $R_2(\mathbf{k}_1, \mathbf{k}_2)$ is similar to a function $w(t)$, where t denotes certain experimentally defined dimensionless combination(s) of relative momentum variables, in 1, 2 or 3 dimensions. For example, the experimental data are frequently represented in one dimension in terms of $t \propto Q_I = \sqrt{-(p_1 - p_2)^2}$, or in three dimensions in terms of $t \propto (Q_0, Q_z, Q_t) = (E_1 - E_2, \mathbf{k}_{1,z} - \mathbf{k}_{2,z}, \sqrt{(\mathbf{k}_{1,x} - \mathbf{k}_{2,x})^2 + (\mathbf{k}_{1,y} - \mathbf{k}_{2,y})^2})$. We define t to be a dimensionless variable, i.e. the relative momentum multiplied with certain spatial scale, e.g. $t = Q_I R_I$.

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The function $w(t)$ can be considered simultaneously as a zero-th order approximation to the function $R_2(t)$, as well as a certain weight function or measure, that is related to an abstract Hilbert space and a complete orthonormal set of functions, $h_n(t)$. This property of the $w(t)$ function will be the basis of the new expansion techique, and the expansion will consist of finding the coefficients of the $h_n(t)$ functions. For the application of the results to expand the real-valued two-particle correlation functions $C_2(\mathbf{k}_1, \mathbf{k}_2)$, we demand that all the $h_n(t)$ functions are real-valued functions of the variable(s) t. We discuss those situations only, when such a complete orthonormal set of functions exists with respect to the measure $d\mu(t) = dt\, w(t)$:

$$\int dt w(t) h_n(t) h_m(t) = \delta_{n,m}, \tag{3}$$

$$f(t) = \sum_{n=0}^{\infty} f_n\, h_n(t), \tag{4}$$

$$f_n = \int dt w(t)\, f(t)\, h_n(t). \tag{5}$$

The Hilbert space H with the measure $dt\, w(t)$ is spanned by the functions $\{h_n(t)\}_{n=0}^{\infty}$.

The important consideration is that we take $w(t)$ to be a function that is simultaneously a measure for an abstract Hilbert space and at the same time, $w(t)$ gives a good approximation to the experimentally measured correlator $R_2(t)$, so that the first few expansion coefficients would suffice to fully describe the measured function. For example, $w(t)$ can be taken as a (multi-dimensional) Gaussian function. In case of a one-dimensional Gaussian function, the Hermite polynomials form a complete orthonormal set of functions with respect to a Gaussian weight. Then we can obtain a convergent expansion in terms of Gaussians and Hermite polynomials as specified in Section 4. Note also that the dimensionless variable t can be defined in one, two or three dimensions, e.g. $t = R_I Q_I$ or $t = (R_L Q_L, R_T Q_T)$ or $t = (R_L Q_L, R_{side} Q_{side}, R_{out} Q_{out})$. The multi-dimensional parameterizations will be addressed explicitly in Section 5.

Let us assume, that the function $g(t) = R_2(t)/w(t)$ is also an element of the Hilbert space H. This is possible, if

$$\int dt\, w(t) g^2(t) = \int dt\, \left[R_2^2(t)/w(t) \right] < \infty, \tag{6}$$

i.e. if the experimentally measured correlator, $R_2(t)$ approaches the value 0 sufficiently fast. With the help of a numerical integration of the experimentally determined $R_2(t)$ function and the selected $w(t)$ zero-th order approximation to $R_2(t)$, the convergence of the integral in the above eq. (6) can be explicitly checked.

The function g can be expanded as

$$g(t) = \sum_{n=0}^{\infty} g_n h_n(t), \tag{7}$$

$$g_n = \int dt\, R_2(t) h_n(t). \tag{8}$$

Thus the expansion coefficients g_n can be determined by a simple numerical evaluation of the integral in eq. (8), using the experimentally determined values for the correlator $R_2(t)$.

From the completeness of the Hilbert space and from the assumption that $g(t)$ is in this Hilbert space, i.e. eq. (6) is satisfied, one obtains the following series expansion for the correlator $R_2(t)$:

$$R_2(t) = w(t) \sum_{n=0}^{\infty} g_n h_n(t). \tag{9}$$

It is obvious from this form, that the best choice of the $w(t)$ function is the form that is as close to the experimentally determined correlator, as possible. For convenience, it is advantageous to choose $h_0(t) = const$ and use the normalization $g_0 h_0(t) = 1$, by an appropriate rescaling of the magnitude of the function $w(t)$. In such a case, the first few expansion coefficients may be sufficient to characterize the correlation function and the correlator $R_2(t)$.

In order to characterize with an independent parameter the strength of the two-particle correlator $R_2(t)$, the expansion-dependent parameter λ_w is introduced, and for convenience, the convention $w(t=0) = 1$ is utilized. Thus the two-particle BECF can be expanded into the following family of series:

$$C_2(t) = \mathcal{N} \left\{ 1 + \lambda_w\, w(t) \sum_{n=0}^{\infty} g_n h_n(t) \right\}, \tag{10}$$

where the core-halo [4] intercept parameter of the correlation function is

$$\lambda_* = \lambda_w \sum_{n=0}^{\infty} g_n h_n(0), \tag{11}$$

and the coefficients of the expansion, g_n can be determined either from numerical evaluation of eq. (8) or from a fit to the measured data points with a particular concrete form of eq. (10).

The above general example can be utilized to study correlation functions in various domains and with various weight functions and related orthogo-

nal polynomials, for example the Legrende, associated Legrende polynomials, spherical harmonics, Gegenbauer, Chebyshev, Hermite, Laguerre or Jacobi polynomials.

In the next two Sections, we demonstrate the power of the method by working out the specific examples for approximately exponential and approximately Gaussian correlators, by using $w(t) = \exp(-t)$ in the $0 \leq t < \infty$ domain and and $w(t) = \exp(-t^2/2)$ in the $-\infty < t < \infty$ domain. As we develop a fitting method, we will utilize only the orthogonality of the complete orthonormal set of functions $\{h_n(t)\}_{n=0}^{\infty}$, giving a simple form for $H_n(t) \propto h_n(t)$ and absorbing the normalization coefficients of $h_n(t)$ into the fit parameters.

3 Laguerre expansion and exponential shapes

If in a zeroth-order approximation the correlation function has an exponential shape, then it is an efficient method to apply the Laguerre expansion, as a special case of eq. (10):

$$t = QR_L, \tag{12}$$
$$w(t) = \exp(-t), \tag{13}$$
$$\int_0^{\infty} dt \, \exp(-t) L_n(t) L_m(t) \propto \delta_{n,m}, \tag{14}$$

where the order-n Laguerre polynomial is defined as

$$L_n(t) = \exp(t)\frac{d^n}{dt^n}(-t)^n \exp(-t). \tag{15}$$

The general form of eq. (10) takes the particular form of the Laguerre expansion [6] as:

$$C_2(Q) = \mathcal{N}\left\{1 + \lambda_L \exp(-QR_L)\left[1 + c_1 L_1(QR_L) + \frac{c_2}{2!}L_2(QR_L) + ...\right]\right\}. \tag{16}$$

The fit parameters are the scale parameters $\mathcal{N}$, λ_L, R_L and the expansion coefficients c_1, c_2, ... The first few Laguerre polynomials are explicitly given as

$$L_0(t) = 1, \tag{17}$$
$$L_1(t) = t - 1, \tag{18}$$

$$L_2(t) = t^2 - 4t + 2, \; .... \tag{19}$$

The Laguerre polynomials are non-vanishing at the origin, hence $C_2(Q = 0) \neq 1 + \lambda_L$. The core-halo intercept parameter, λ_*, is defined by the extrapolated value of the two-particle correlation function at $Q = 0$, see refs. [4,12–14] for further details. If the core/halo model is applicable, and effects of partial coherence in the particle emitting source can be neglected, the core-halo intercept parameter λ_* measures the squared fraction of bosons emitted directly from the core, an important physical observable that is related to the degree of partial restoration of $U_A(1)$ symmetry in hot and dense hadronic matter [2,3], as a partial $U_A(1)$ symmetry restoration leads to enhanced production of η' mesons and suppression of the core fraction of directly produced pions at low values of transverse momentum.

The physically significant core-halo intercept parameter λ_*, and its error $\delta\lambda_*$ can be obtained from the parameter λ_L of the Laguerre expansion as

$$\lambda_* = \lambda_L[1 - c_1 + c_2 - ...], \tag{20}$$
$$\delta^2\lambda_* = \delta^2\lambda_L\left[1 + c_1^2 + c_2^2 + ...\right] + \lambda_L^2\left[\delta^2c_1 + \delta^2c_2 + ...\right]. \tag{21}$$

Any Bose-Einstein correlation function can be expanded into a *convergent* Laguerre expansion of the form of eq. (16), provided that

$$\int\limits_0^\infty dt\, R_2^2(t)\exp(+t) < \infty, \tag{22}$$

where $t = QR_L$ stands for the dimensionless scale variable. In principle, the left-hand side of this unequality can be evaluated directly from the experimental data.

4 Edgeworth expansion and Gaussian shapes

If, in a zeroth-order approximation, the correlation function has a Gaussian shape, then it is an efficient method to apply the Edgeworth expansion to characterize the deviation from the Gaussian form in the following manner:

$$t = \sqrt{2}QR_E, \tag{23}$$
$$w(t) = \exp(-t^2/2), \tag{24}$$
$$\int\limits_{-\infty}^\infty dt\, \exp(-t^2/2)H_n(t)H_m(t) \propto \delta_{n,m}, \tag{25}$$

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where the order-n Hermite polynomial is defined as

$$H_n(t) = \exp(t^2/2) \left(-\frac{d}{dt}\right)^n \exp(-t^2/2). \tag{26}$$

The general form of eq. (10) takes the particular form of the Edgeworth expansion [15,5,6] as:

$$C_2(Q) = \mathcal{N} \left\{ 1 + \lambda_E \exp(-Q^2 R_E^2) \times \right.$$
$$\left. \left[1 + \frac{\kappa_3}{3!} H_3(\sqrt{2}QR_E) + \frac{\kappa_4}{4!} H_4(\sqrt{2}QR_E) + ... \right] \right\}. \tag{27}$$

The fit parameters are the scale parameters $\mathcal{N}$, λ_E, R_E and the expansion coefficients κ_3, κ_4, ... , that coincide with the cumulants of rank 3, 4, ..., of the correlation function. The first few Hermite polynomials are listed as

$$H_1(t) = t, \tag{28}$$
$$H_2(t) = t^2 - 1, \tag{29}$$
$$H_3(t) = t^3 - 3t, \tag{30}$$
$$H_4(t) = t^4 - 6t^2 + 3, ... \tag{31}$$

The physically significant core-halo intercept parameter λ_* can be obtained from the Edgeworth fit of eq. (27) as

$$\lambda_* = \lambda_E \left[1 + \frac{\kappa_4}{8} + ... \right], \tag{32}$$
$$\delta^2 \lambda_* = \delta^2 \lambda_E + (\kappa_4 \delta \lambda_E + \lambda_E \delta \kappa_4)^2/64 + ... \tag{33}$$

Any Bose-Einstein correlation function can be expanded into a *convergent* Edgeworth expansion, if

$$\int\limits_{-\infty}^{\infty} dt \, R_2^2(t) \exp(+t^2/2) < \infty. \tag{34}$$

This latter requirement can be checked experimentally, if necessary. This expansion technique was applied in the conference contributions [5,6] to the AFS minimum bias and 2-jet events to characterize successfully the deviation of data from a Gaussian shape. It was also successfully applied to characterize the non-Gaussian nature of the correlation function in two-dimensions in case of the preliminary E802 data in ref. [6], and it was recently applied to characterize the non-Gaussian nature of the three-dimensional two-pion BECF in $e^+ + e^-$ reactions at LEP - 1 [16].

5 Multi-dimensional expansions

Genuine multi-dimensional expansions can be obtained by using multi-dimensional complete orthogonal sets of functions. The simplest two-dimensional example is the study of angular correlations on the surface of the unit sphere (e.g. to study correlations of jets within jets in QCD). In this case, $t = (\theta, \phi)$, the measure is $w(t) = \sin(\theta)$ and the domain is $(0 \leq \theta \leq \pi,\ 0 \leq \phi < 2\pi)$. The orthogonal polynomials are the well-known spherical harmonics $Y_l^m(\theta, \phi)$ that satisfy

$$\int_0^\pi d\theta \sin(\theta) \int_0^{2\pi} d\phi \, \overline{Y}_k^n(\theta, \phi) Y_l^m(\theta, \phi) \propto \delta_{k,l} \delta_{m,n} \tag{35}$$

and the angular correlations can be expanded in a series as

$$C_2(\theta, \phi) = 1 + \lambda_Y \sin(\theta)[1 + \sum_{l=1}^{\infty} \sum_{m=-l}^{l} c_{lm} Y_l^m(\theta, \phi)]. \tag{36}$$

This example should be sufficient to demonstrate how the method can be generalized to higher dimensions. A systematic treatise of the multi-dimensional realizations of the general expansion method as described in Section 2 is beyond the scope of the present paper.

In the following we consider the special case of factorized multi-dimensional distributions. A more profound technique can be obtained based on the specialization of the general arguments of section 2 to the case of multi-dimensional BECF-s. The two-dimensional Edgeworth expansion and the interpretation of its parameters is described in the handbook on mathematical statistics by Kendall and Stuart [17].

If the two-particle BECF can be factorized as a product of (two or more) functions of one variable each, then e.g. a Laguerre or an Edgeworth expansion can be applied to the multiplicative factors – functions of one variable, separately. This method was applied recently to study the non-Gaussian features of multi-dimensional Bose-Einstein correlation functions in refs. [5,16].

Let us restrict ourselves to the situation when the multi-dimensional Fourier-transformed emission function factorizes in certain set of variables. For example, such a boost-invariant form of factorization is assumed in the Buda-Lund parameterization of the two-particle Bose-Einstein correlation functions in refs. [18,19].

A simple expression for the two-particle correlation functions is obtained for longitudinally approximately boost-invariant, cylindrically symmetric particle emitting sources, in the Buda-Lund picture, if we assume that the emission function factorizes as a product of an effective proper-time distribution, a space-time rapidity distribution and a transverse coordinate distribution where $\eta = 0.5 \log[(t + r_z)/(t - r_z)]$ is the space-time rapidity, $\tau = \sqrt{t^2 - r_z^2}$ is the longitudinally boost-invariant proper-time distribution [18,19].

With the help of a small source size (or large relative momentum) expansion, the two-particle Bose-Einstein correlation function can be written into the following boost-invariant, factorized Buda-Lund form:

$$
C_2(\Delta k, K) = 1 + \lambda_*(K) \frac{|\tilde{H}_*(Q_=)|^2}{|\tilde{H}_*(0)|^2} \frac{|\tilde{G}_*(Q_\parallel)|^2}{|\tilde{G}_*(0)|^2} \frac{|\tilde{I}_*(Q_\perp)|^2}{|\tilde{I}_*(0)|^2}. \tag{37}
$$

Here, the Fourier-transformed proper-time , space-time rapidity and transversal coordinate distributions, $\tilde{H}_*(Q_=)$, $\tilde{G}_*(Q_\parallel)$ and $\tilde{I}_*(Q_\perp)$ can be of power-law, exponential, Gaussian or other types, corresponding to the underlying space-time structure of the particle emitting source, hence it is natural to apply the one-dimensional Edgeworth or the Laguerre expansions to any of these factors.

The invariant *temporal*, *parallel* and *perp*endicular relative momentum components are defined, respectively, as

$$
Q_= = Q_0 \cosh[\overline{\eta}] - Q_z \sinh[\overline{\eta}] \equiv Q \cdot \overline{n}, \tag{38}
$$
$$
Q_\parallel = Q_0 \sinh[\overline{\eta}] - Q_z \cosh[\overline{\eta}] \equiv Q \times \overline{n}, \tag{39}
$$
$$
Q_\perp = \sqrt{Q_x^2 + Q_y^2}, \tag{40}
$$
$$
Q^2 = Q \cdot Q = (Q_=)^2 - (Q_\parallel)^2 - Q_\perp^2. \tag{41}
$$

In the above, $a \cdot b = a^\mu b_\mu = a_0 b_0 - \mathbf{ab} = a_0 b_0 - a_x b_x - a_y b_y - a_z b_z$ stands for the inner product of four-vectors. The direction of the normal-vector $\overline{n}$ is characterized as $\overline{n}^\mu = (\cosh[\overline{\eta}], 0, 0, \sinh[\overline{\eta}])$, by a mean space-time rapidity $\overline{\eta}$ [18,19] in the LCMS frame [20]. Hence this direction is boost-invariant and $\overline{\eta}$ is one of the fit parameters. As $\overline{n} \cdot \overline{n} = +1$, this normal-vector points into a time-like direction and $Q \cdot \overline{n} = Q_=$ is an invariant time-like component of the relative momentum. The relative momentum component $Q_\parallel$ is parallel to the longitudinal direction (which is the thrust axis in jet physics, or the beam direction in heavy ion physics), and $Q_\parallel$ is invariant to boosts along this direction. Finally, $Q_\perp$ is the remaining perpendicular (or transverse) component of the relative momentum, it is also invariant for longitudinal boosts.

In eq. (37) the Fourier-transformed distributions are defined and explained in greater detail in refs. [18,19]. Although these distributions can be theoretically evaluated from an assumed shape of the emission function $S(x, k)$, for example see ref [18,19], here we are interested in the model-independent characterization of the Bose-Einstein correlation functions, exclusively.

From a three-dimensional analysis of two-pion Bose-Einstein correlation data at LEP1 [16] we know that the BECF is approximately a factorized Gaussian. Thus, a good candidate to characterize the non-Gaussian nature of these correlations in 3 dimensions, in a boost-invariant manner, is the following Edgeworth expansion of the factorized form of the Buda-Lund correlation function:

$$C_2(\Delta k, K) = 1 + \lambda_E \exp(-Q_=^2 R_=^2 - Q_{||}^2 R_{||}^2 - Q_\perp^2 R_\perp^2) \times$$
$$\left[1 + \frac{\kappa_{3,=}}{3!} H_3(\sqrt{2} Q_= R_=) + \frac{\kappa_{4,=}}{4!} H_4(\sqrt{2} Q_= R_=) + ... \right] \times$$
$$\left[1 + \frac{\kappa_{3,||}}{3!} H_3(\sqrt{2} Q_{||} R_{||}) + \frac{\kappa_{4,||}}{4!} H_4(\sqrt{2} Q_{||} R_{||}) + ... \right] \times$$
$$\left[1 + \frac{\kappa_{3,\perp}}{3!} H_3(\sqrt{2} Q_\perp R_\perp) + \frac{\kappa_{4,\perp}}{4!} H_4(\sqrt{2} Q_\perp R_\perp) + ... \right]. \quad (42)$$

What are the fit parameters in eq. (42)? We have 5 *free scale parameters* for cylindrically symmetric, longitudinally expanding sources, and *three series of shape parameters*. The scale parameters are λ_E, $R_=$, $R_{||}$, $R_\perp$ and $\overline{\eta}$, that characterize the effective source at a given mean momentum, by giving the vertical scale of the correlations, the invariant temporal, longitudinal and transverse extensions of the source and its invariant direction, which is the space-time rapidity of the effective source in the LCMS frame (the frame where $k_{1,z} + k_{2,z} = 0$, [20]). The three series of shape parameters are $\kappa_{3,=}$, $\kappa_{4,=}$, ... , $\kappa_{3,||}$, $\kappa_{4,||}$, ... , $\kappa_{3,\perp}$, $\kappa_{4,\perp}$, ...

In principle, each of these parameters may depend on the mean momentum $\mathbf{K}$. At any fixed value of the mean momentum $\mathbf{K}$, the above free parameters of the invariant Buda-Lund correlation function can be fitted to data, without any a priori assumption on the shape of the source emission function, based only on the cylindrical symmetry and the boost-invariant factorization property of the particle emitting source.

6   Application of the results

As the Laguerre and the Edgeworth expansions of Bose-Einstein correlation functions were introduced with quite some success in refs. [5,6] to characterize

the AFS 2-jet and the AFS minimum-bias data in 1 dimension, to characterize the non-Gaussian features of the E802 preliminary $Si + Au \rightarrow \pi^+ + \pi^+ + X$ data in two dimensions [6], and they were reported to successfully characterize the non-Gaussian features of the L3 two-pion BECF at LEP1 in two and three dimensions, [16] (although using a non-invariant formulation), we think that there is no need to further demonstrate the applicability of the Edgeworth expansion and its multi-dimensional generalizations in correlation studies in high energy physics.

However, the applications of the Laguerre expansions were not yet presented in the literature as far as we know. Hence, let us demonstrate in Figure 1 the power of the Laguerre expansions to characterize two well-known, non-Gaussian correlation functions: the $C_2(Q_I)$ correlation functions as determined by the UA1 and the NA22 experiments [7,8]. Note that the invariant momentum difference Q^2 is binned in logarithmic scale, and the resolution of both experiments goes down to about 30 MeV, a very small scale as compared to the typical 200 MeV half-width of the BECF in particle physics.

We note, that Fig. 1. is just an illustration of the power of Laguerre expansions, however, the authors do not intend to replace the power-law description of these correlation functions by the Laguerre expansion, and the authors are aware of the fact that a power-law fit gives a similarly good χ^2/NDF with smaller number of fit parameters.

Table 1

Best fits to UA1 and NA22 two-particle correlations using a Laguerre expansion

Parameter	UA1		NA22	
	Value	Error	Value	Error
$\mathcal{N}$	1.355	$\pm\ 0.003$	0.95	$\pm\ 0.01$
λ_L	1.23	$\pm\ 0.07$	1.37	$\pm\ 0.10$
R_L [fm]	2.44	$\pm\ 0.12$	1.35	$\pm\ 0.14$
c_1	0.52	$\pm\ 0.03$	0.63	$\pm\ 0.06$
c_2	0.45	$\pm\ 0.04$	0.44	$\pm\ 0.06$
χ^2/NDF	$41.2/41 = 1.01$		$20.0/34 = 0.59$	

From Table 1, one can determine that the quality of the Laguerre expansion fits is statistically acceptable both in case of the UA1 and in case of the NA22 data. We can also determine the core-halo model intercept parameter, $\lambda_* = 1.14 \pm 0.10$ (UA1) and $\lambda_* = 1.11 \pm 0.17$ (NA22). As both of these values are within errors equal to unity, the maximum of the possible value of the intercept parameter λ_* in a fully chaotic source, we conclude that either there are other than Bose-Einstein short-range correlations observed by both collaborations, or in case of this measurement the full halo of long-lived resonances is

resolved [4,13]. This is very interesting and the distribution of long-lived resonance fractions and decay times may be related to the approximate power-law shape of the fitted correlation functions [22]. The full resolution of the halo is theoretically possible only because the η'-s and η-s were either produced in a negligible number, or because their effects were corrected for in the Monte-Carlo evaluation of the background distribution.

7 Summary

In this paper we described a general method to characterize, in a model and theory independent manner the shape of the two-particle Bose-Einstein correlation functions with the help of mutually orthogonal sets of functions, assuming that the measure of the abstract Hilbert space can be identified with the approximate shape of the correlation function. For one-dimensional, approximately Gaussian correlation data, the Edgeworth expansion technique is obtained. For one-dimensional, approximately exponential shapes, we obtained the expansion in terms of the Laguerre polynomials. We generalized these expansion techniques to two and three dimensions. We also demonstrated that the method works well: we characterized the deviation of the two-particle Bose-Einstein correlation function of the UA1 and the NA22 collaborations from the exponential shape with the first two Laguerre coefficients.

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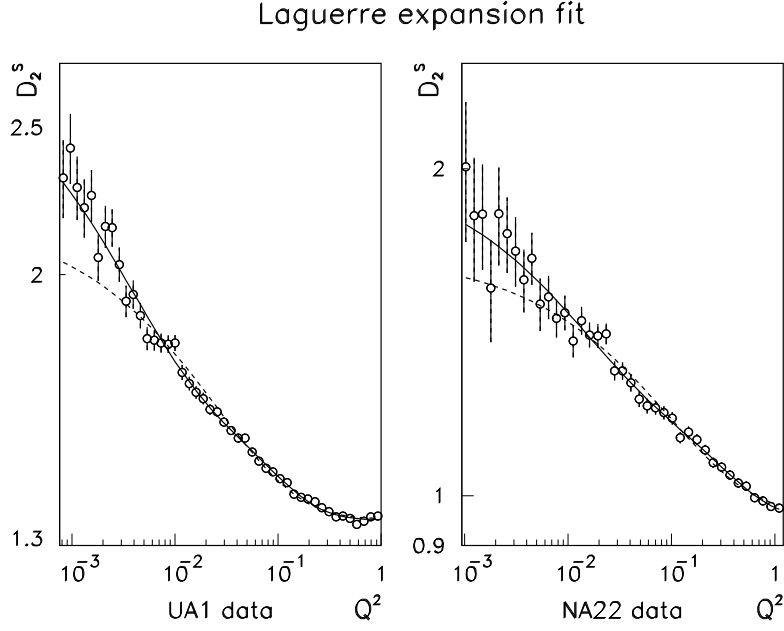
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Fig. 1. The figures show D_2^s which is proportional to the two-particle Bose-Einstein correlation function, as measured by the UA1 and the NA22 collaborations. The dashed lines stand for the exponential fit, which clearly underestimates the measured points at low value of the squared invariant momentum difference Q_I^2 (note the logarithmic horizontal scale). The solid lines stand for the fits with the Laguerre expansion method, which is able to reproduce the data with a statistically acceptable χ^2/NDF. The fit results are summarized in Table 1.